DYNAMIC ANALYSIS OF FGM NANOBEAMS UNDER MOVING LOAD CONSIDERING SHEAR DEFORMATION EFFECT

A. ELMEICHE, M. BOUAMAMA, A. MEGUENI

Abstract—This investigation focuses on the dynamic behavior analysis of functionally graded materials (FGM) nanobeams excited by a moving concentrated load, based on the nonlocal elasticity theory, taking into account the shear deformation beams effect. The government equations of motion are modeled by introducing weak forms into the forced vibratory system, under various orders beams theories, while including rotational inertia. The mechanical properties of FGMs nanobeams vary continuously through the thickness direction according to the power-law exponent form. Rayleigh-Ritz solutions are employed and combined with Newmark's method to find dynamic vibration analysis responses. A convergence study is established and numerical results are validated with those available in the literature to show the reliability and efficiency of current model. Several examples are discussed and examined to determine the impact of the nonlocal parameter, material distribution, shear deformation beam effect, slenderness ratio and the velocity of the moving constant load on transverse dynamic responses of FGM nanobeams.

Index Terms—Dynamic analysis, FGM, nanobeams, moving load, shear deformation, nonlocal elasticity, velocity, dynamic responses.

1 Introduction

Nowadays, Nanosciences and nanotechnologies are booming thanks to the development of new tools, observation and analysis. The term "nano" refer to the nanometer scale and more widely for the clearly submicron dimensions [1]. Nanoscience is, more simply, the study of the fundamental principles of molecules and structures with at least a dimension of about 1 to 100 nanometers. These structures are known as nanostructures [2].

Nanoscience and nanotechnology cover a much broader field of applications with nanomaterials that play a key role in many areas, such as automotive, aerospace, building, packaging, tribology, catalysis, environment, etc. In this regard, certain types of nanostructures such as nanobeam, nanoplate and nanotube have been developed for use in modern technologies such as Electrical Devices and Atomic Force Microscopes.

Recently, the dynamic behavior of structural elements with FGMs is of considerable importance in the fields of research and industry. Typically, these materials are made from a mixture of metal and ceramic, or a combination of materials. The ceramic component provides a high temperature because of its low resistance to thermal conductivity. FGMs are used in very different engineering applications such as automotive, aerospace, defense, and more recently electronics, nuclear reactors, biomedical and transportation.

Most researchers are interested in the static and dynamic study of nanostructures. Works based on Eringen's nonlocal theory have been published, Eringen, 2002 [3]; Thai, 2011 [4]; and Aydogdu, 2009 [5]. Others were used in the vibratory study, Wang et al, 2011[6] and Eltaher et al ., 2013 [7] used the classical Euler-Bernoulli theory in the vibratory study of nanobeams.

Moreover other researchers, Wu et al., 2011 [8]; Junghorban, 2011 [9]; Rahmani and Pedram, 2014 [10] used Timoshenko's theory in the study of nanobeam. Lei et al., 2013 [11] studied the Kelvin-Voigt viscoelastic damping by Timoshenko's theory. Other teams have studied the thermal effect on natural frequencies of orthopedic nano-plates, Satish et al., 2012 [12].

In recent years, nanostructures have attracted a lot of attention in terms of research on forced vibratory behavior. Recent research on dynamic response has been done: Kiani, 2010 [13] investigated the dynamic response of a single-walled carbon nanotube (SWCNT) under a moving nanoparticle based on nonlocal theory. Simsek, 2010 [14] studied the forced vibration of a single-walled carbon nanotube under a mobile harmonic load. Simsek, 2011 [15] determinate the nonlocal effects in the forced vibration of a dual carbon nanotube system under a moving nanoparticle. Mehrdad, 2015 [16] analyzed the forced transversal vibrations of a closed-walled double-walled carbon nanotube system containing a fluid with the effect of the compression of the axial load. Hosseini et al., 2017 [17] and Lei et al., 2016 [18] have studied the dynamic responses of nanobeams in FGM subjected to a constant moving load based on the classical Euler-Bernoulli theory.

The main objective in this research is to present an analysis on the dynamic behavior of functionally graded materials (FGM) nanobeams subjected to a dynamic punctual constant transversal load that moves by a transient motion, taking into account the effect of shear deformation beams effect. Based on the Eringen's nonlocal constitutive relations, the government

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equations of motion are derived using Hamilton principle by introducing the weak forms into the system. Forced vibrations are modeled for all order beam theories, classical, the first-order and higher-order shear deformation beam theories while including rotary inertia. The material properties of FGMs nanobeams vary continuously in the thickness direction according to the power law exponent form. Rayleigh-Ritz solutions are adopted to discretize the spatial partial derivatives of the system where the displacement components of the nanobeams cross section are expressed in a series of simple algebraic polynomials and dynamic vibration analysis responses are also solved numerically using Newmark's temporal integration method. To show the reliability and precision for present model, the dynamic responses obtained in the vibratory analysis are converged towards satisfactory results and validated by comparison with those available in the literature. Several examples are treated and illustrated in graphical and tabular form. In this study, the influence of the nonlocal parameter, material distribution, shear deformation beam effect, slenderness ratio and the velocity of the mobile constant load on the transverse dynamic responses of the FGM nanobeams are examined and discussed in detail.

2 Modeling of FGM nanobeams system

Consider a nanobeam in FGM of length "L", width "b" and thickness "h" subjected to a concentrated transverse force "P" which moves in a transient motion defined by a speed "vp". It is assumed that the nanobeam has a linear elastic behavior with Cartesian coordinate system (O, x, y, z) as shown in Figure 1.

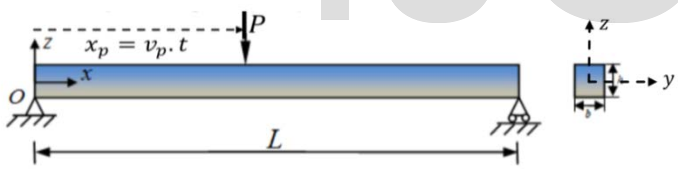

FIG.1. FGM NANOBEAM SCHEME UNDER A DYNAMIC MOVING LOAD

The functionally graded material (FGM) is composed of two different extreme materials. The material properties: Young's modulus (E), Poisson's ratio (υ) and the density (ρ), vary continuously in the thickness direction "h", according to a function of the volume fractions. Based on the rule of the mixture, the effective material properties (P) can be written as:

$$P = P_U V_U + P_L V_L \tag{1}$$

P_U, P_L, V_U and V_L are the corresponding material properties and the volume fractions of the upper and the lower surfaces of the nanobeam bound by:

$$P_U + V_L = 1 \tag{2}$$

In this study, the FGM profile of the upper volume fraction is assumed to follow the power law form that is written by Wakashima [19]:

$$V_U = \left(\frac{z}{h}+\frac{1}{2}\right)^k \tag{3}$$

(k) is the power law index, non-negative constant $(0 \le k \le \infty)$, which determines the mixing law variation along the thickness of the nanobeam, as shown in Figure 2. Using equation (1), (2) and (3), the effective materials properties (P) of the FGM nanobeam are expressed as follows:

$$P(z) = (P_U - P_L)\left(\frac{z}{h}+\frac{h}{2}\right)^k + P_L \tag{4}$$

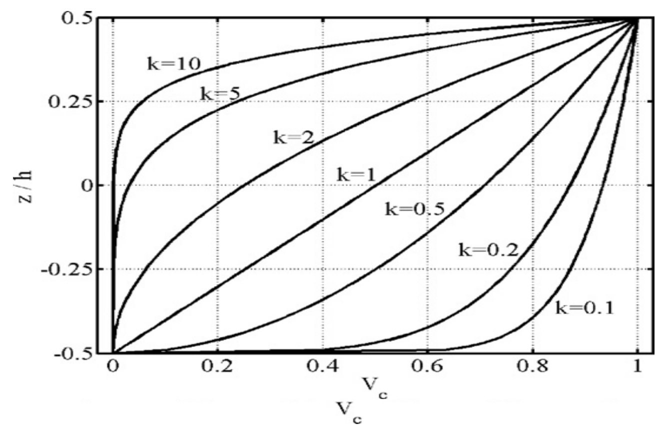

FIG.2. UPPER VOLUME FRACTION PROFILE (VU) THROUGH THE THICKNESS OF FGM NANOBEAM

3 Nonlocal beam THEORY

The response of nanoscale structures is different from classical theory. According to the nonlocal elasticity beam theory, the stress field at an arbitrary point "x" in an elastic continuum depends not only on the stress field at the same point, but also on the stress at all other parts of the body [20]. This assumption can be expressed as follows:

$$[1-\mu\nabla^2]\overline{\sigma} = \overline{C} : \overline{\varepsilon} \tag{5}$$

Where $\mu = (e_0 a)^2$ is the nonlocal parameter, (e_0) is a constant appropriate to each material, (a) is the internal characteristic length [21]. When (a) is zero, we can derive the constitutive relation of the classical beam theory. ∇^2 is the Laplacian operator and double dot tensor product. $(\overline{\sigma})$ is the stress tensor, $(\overline{C})$ is the Hookean elasticity tensor, and $(\overline{\varepsilon})$ is the strain tensor. The general nonlocal constitutive relationships for nanobeams are written as follows:

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{xz}\end{bmatrix} - \mu\frac{\partial^2}{\partial x^2}\begin{bmatrix}\sigma_{xx}\\ \sigma_{xz}\end{bmatrix} = \begin{bmatrix}Q_{11} & 0\\ 0 & Q_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \gamma_{xz}\end{Bmatrix} \tag{6}$$

The reduced elastic constants are defined by:

$$Q_{11} = \frac{E(z)}{(1-\upsilon(z)^2)} \quad \text{and} \quad Q_{55} = \frac{E(z)}{2.(1+\upsilon(z))} \tag{7}$$

As $E(z)$, $\upsilon(z)$ are the Young's modulus and the Poisson's ratio respectively, according to the thickness direction (z). σ_{xx} is the axial normal stress. σ_{xz} is the shear stress. ε_{xx} is the axial deformation and γ_{xz} is the shear deformation. To study the nonlocal effect on the nanobeams behavior, the scale coefficient (μ) is proposed between 0 and 4 [4].

4 Mathematical Development

Based on the general shear deformation beam theory, the displacements coordinates of any point of the nanobeam are given as:

$$\begin{cases} u(x,z,t) = u_0(x,t) - z.w_{0,x}(x,t) + f(z).\varphi_0(x,t) \\ v(x,z,t) = 0 \\ w(x,z,t) = w_0(x,t) \end{cases} \quad (8)$$

$u_0 = (x,t)$ and $w_0 = (x,t)$ are the displacement components in the middle of the section and on the mean line of the FGM nanobeam respectively along the (x) and (z) axes. $\varphi_0 = (x,t)$ is the distortion, also measured on the middle line of the FGM nanobeam. (t) represent the time index. $f(z)$ is the shape function which characterizes the transverse shear and stress distribution along the thickness direction (z). Various order beams theories are used in this study:

Classical beam theory (CBT): $f(z) = 0$

First order shear deformation beam theory (FSDBT): $f(z) = z$

High order shear deformation beam theory [22] (PSDBT):

$$f(z) = z.\left(1 - \frac{4z^2}{3h^2}\right)$$

The shear correction factor is considered as ks = 5/6 for FSDBT.

In the small disturbances hypothesis, the strain-displacement relations of the general beam theories are written as follows:

$$\begin{cases} \varepsilon_{xx} = \dfrac{\partial u(x,z,t)}{\partial x} = u_{0,x} - zw_{0,xx} + f(z)\varphi_{0,x} \\ \gamma_{xz} = \dfrac{\partial u(x,z,t)}{\partial z} + \dfrac{\partial w(x,z,t)}{\partial x} = f'(z)\varphi_0 \end{cases} \quad (9)$$

The equilibrium equations will be obtained from the Hamilton principle:

$$\int_{t_1}^{t_2} (\delta K - (\delta S + \delta V)).\mathrm{dt} = 0 \quad (10)$$

The virtual strain energy δS :

$$\delta S = \int_V \sigma_{ij}.\delta\varepsilon_{ij} dV = \int_V \sigma_{xx}\delta\varepsilon_{xx} dV + \int_V \sigma_{xz}\delta\gamma_{xz} dV \quad (11)$$

$$\delta S = \int_0^L N^c.\delta u_{0,x}.dx - \int_0^L M^c.\delta w_{0,xx}.dx + \int_0^L M^{sd}.\delta\varphi_{0,x}.dx + \int_0^L Q.\delta\varphi_0.dx \quad (12)$$

N^c, M^c, M^{sd} and Q are the stress resultants defined by:

$$(N^c, M^c, M^{sd}, Q) = \int_A \sigma_{xx}.(1, z, f(z), f'(z)).\mathrm{dA} \quad (13)$$

The results indicated with an exponent (c) are the conventional ones of the classical beams theory, while the others with exponent (sd) are additional quantities incorporating shear deformation effects. By substituting the stress-strain relations into the definitions of the force and the moment resultants from the present theory, we obtain the following constitutive equations:

$$\begin{bmatrix} N^c \\ M^c \\ M^{sd} \end{bmatrix} = \begin{bmatrix} A_{11} & B_{11} & E_{11} \\ B_{11} & D_{11} & F_{11} \\ E_{11} & F_{11} & H_{11} \end{bmatrix} \begin{bmatrix} u_{0,x} \\ -w_{0,xx} \\ \varphi_{0,x} \end{bmatrix} \text{ and } Q = [A_{55}][\varphi_0] \quad (14)$$

The extensional, coupling, bending and transverse shear stiffnesses are given as follows:

$$(A_{11}, B_{11}, D_{11}) = b\int_{-\frac{h}{2}}^{+\frac{h}{2}} Q_{11}(1, z, z^2)\,\mathrm{dz} \quad (15)$$

$$(\mathrm{E}_{11}, \mathrm{F}_{11}, \mathrm{H}_{11}) = b\int_{-\frac{h}{2}}^{+\frac{h}{2}} Q_{11}\,\mathrm{f(z)(1, z, f(z))}\,\mathrm{dz} \quad (16)$$

$$A_{55} = k_s b\int_{-\frac{h}{2}}^{+\frac{h}{2}} Q_{55}[\mathrm{f}'(\mathrm{z})]^2\,\mathrm{dz} \quad (17)$$

The shear correction factor is considered as ks = 5/6 for FSDBT.

Virtual kinetic energy δK :

$$\delta K = \int_V \rho(z)\dot{u}.\delta\dot{u}.dV + \int_V \rho(z)\dot{w}.\delta\dot{w}.dV \quad (18)$$

$$\delta K = \int_0^L (I_1.\ddot{u}_0 - I_2.\ddot{w}_{0,x} + I_3.\ddot{\varphi}_0).\delta u_0.dx - \int_0^L (I_2.\ddot{u}_0 - I_4.\ddot{w}_{0,x} + I_5.\ddot{\varphi}_0).\delta w_{0,x}.dx + \int_0^L (I_3.\ddot{u}_0 - I_5.\ddot{w}_{0,x} + I_6.\ddot{\varphi}_0).\delta\varphi_0.dx + \int_0^L I_1.\ddot{w}_0.\delta w_0.dx \quad (19)$$

Such as:

$$(I_1, I_2, I_3, I_4, I_5, I_6) = \int_A \rho(z).(1, z, f(z), z^2, zf(z), f(z)^2).dA \quad (20)$$

The virtual potential energy δV of the transverse load in motion:

$$\delta V = \int_0^L \mathrm{P}(x,t).\delta w_0.dx \quad (21)$$

The constant moving load $P(x,t)$ can be defined:

$$P(x,t) = -P.\delta(x - v_p.t) \quad (22)$$

Where $\delta(.)$ is Dirac delta function, (P) is concentrated load amplitude, (υ_n) is the moving load speed.

By replacing the equations (12), (19) and (21) in equation (10), integrating by parts and obtaining the following equilibrium equations:

$$\frac{\partial N^c}{\partial x} = I_1.\ddot{u}_0 - I_2.\ddot{w}_{0,x} + I_3.\ddot{\varphi}_0 \quad (23)$$

$$\frac{\partial^2 M^c}{\partial x^2} = I_2.\ddot{u}_{0,x} - I_4.\ddot{w}_{0,xx} + I_5.\ddot{\varphi}_{0,x} + I_1.\ddot{w}_0 + P.\delta(\mathrm{x} - v_p.t) \quad (24)$$

$$\frac{\partial M^{sd}}{\partial x} - Q = I_3.\ddot{u}_0 - I_5.\ddot{w}_{0,x} + I_6.\ddot{\varphi}_0 \quad (25)$$

The Rayleigh-Ritz solution is adopted to discretize the partial derivatives of components displacement for the forced vibratory system and the shape functions are developed in terms of the algebraic polynomial series as indicated by the following formulas:

$$u_0(x,t) = \sum_{j=1}^{n} \varphi_j(x).u_j(t);$$

$$w_0(\mathrm{x,t}) = \sum_{k=1}^{n} \psi_k(x).w_k(t);$$

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$\varphi_0(x,t)=\sum_{p=1}^{n}\phi_p(x).\varphi_p(t)$;

$\varphi_j(x)=(1-x)^{q_0}.x^{(j+p_0)-1}$;

$\psi_k(x)=(1-x)^{q_0}.x^{(k+p_0)-1}$;

$\phi_p(x)=(1-x)^{q_0}.x^{(p+p_0)-1}$;

Where $u_i(t)$, $w_k(t)$ and $\varphi_p(t)$ are the Ritz's temporary coefficients. $\varphi_i(x)$, $\psi_k(x)$ and $\phi_p(x)$ are the Ritz approximations, which must satisfy the boundary conditions. (n) is the number of polynomials involved in the admissible functions. (p_0, q_0) are the indices of Rayleigh-Ritz test function, they depend on the boundary conditions of FGM nanobeams [23].

By introducing the equilibrium equations (23, 24 and 25) in the nonlocal constitutive relation of Eringen (6), we obtain the following differential equations:

$$\left(1-\mu\frac{\partial^2}{\partial x^2}\right).(I_1.\ddot{u}_0-I_2.\frac{\partial \ddot{w}_0}{\partial x}+I_3.\ddot{\varphi}_0)-\frac{\partial}{\partial x}(A_{11}.\frac{\partial u_0}{\partial x}-B_{11}\frac{\partial^2 w_0}{\partial x^2}+E_{11}.\frac{\partial \varphi_0}{\partial x})=0 \quad (26)$$

$$\left(1-\mu\frac{\partial^2}{\partial x^2}\right).\left(I_1.\ddot{w}_0+\frac{\partial}{\partial x}(I_2\ddot{u}_0-I_4.\frac{\partial \ddot{w}_0}{\partial x}+I_5.\ddot{\varphi}_0)\right)-\frac{\partial^2}{\partial x^2}(B_{11}.\frac{\partial u_0}{\partial x}-D_{11}\frac{\partial^2 w_0}{\partial x^2}+F_{11}.\frac{\partial \varphi_0}{\partial x})=-\left(1-\mu\frac{\partial^2}{\partial x^2}\right).(P.\delta(x-v_p t)) \quad (27)$$

$$\left(1-\mu\frac{\partial^2}{\partial x^2}\right).(I_3.\ddot{u}_0-I_5.\ddot{w}_{0,x}+I_6.\ddot{\varphi}_0)-\frac{\partial}{\partial x}(E_{11}.\frac{\partial u_0}{\partial x}-F_{11}\frac{\partial^2 w_0}{\partial x^2}+H_{11}.\frac{\partial \varphi_0}{\partial x})+A_{55}.\varphi_0=0 \quad (28)$$

After performing the integration by part on equations (26), (27) and (28) with weighted functions respectively, $\psi_i(x)$ and $\emptyset_i(x)$ (i = 1,2, ...n), which must satisfy the boundary conditions, the weak forms of motion government equations, which equates to both the ordinary differential equations, can be written in the following final form:

$$\int_0^L \varphi'_i(x).\left(A_{11}\sum_{j=1}^{n}\varphi'_j(x).u_j(t)-B_{11}\sum_{k=1}^{n}\psi''_k(x).w_k(t)+E_{11}\sum_{p=1}^{n}\phi'_p(x).\varphi_p(t)\right).dx+$$
$$\int_0^L \varphi_i(x)\left(I_1\sum_{j=1}^{n}\left(1-\mu\frac{d^2}{dx^2}\right)\varphi_J(x).\ddot{u}_j(t)-I_2\sum_{k=1}^{n}\left(1-\frac{d^2}{dx^2}\right)\psi'_k(x).\ddot{w}_k(t)+I_3\sum_{p=1}^{n}\left(1-\mu\frac{d^2}{dx^2}\right)\phi_p(x).\ddot{\varphi}_p(t)\right).dx-[N^c.\varphi_i(x)]_0^L=0 \quad (29)$$

$$\int_0^L \psi''_i(x).\left(-B_{11}\sum_{j=1}^{n}\varphi'_j(x).u_j(t)+D_{11}\sum_{k=1}^{n}\psi''_k(x).w_k(t)-F_{11}\sum_{p=1}^{n}\phi'_p(x).\varphi_p(t)\right).dx$$
$$+\int_0^L \psi'_i(x).\left(-I_2\sum_{j=1}^{n}\left(1-\mu\frac{d^2}{dx^2}\right)\varphi_j(x).\ddot{u}_j(t)+I_4\sum_{k=1}^{n}\left(1-\mu\frac{d^2}{dx^2}\right)\psi'_k(x).\ddot{w}_k(t)-I_5\sum_{p=1}^{n}\left(1-\mu\frac{d^2}{dx^2}\right)\phi_p(x).\ddot{\varphi}_p(t)\right).dx \quad (30)$$
$$+\int_0^L \psi_i(x)\left(I_1\sum_{j=1}^{n}\left(1-\mu\frac{d^2}{dx^2}\right)\psi_k(x).\ddot{w}_k(t)\right).dx+\left[M^c.\psi'_i(x)\right]_0^L-\left[V^c.\psi_i(x)\right]_0^L=-\int_0^L \psi_i(x).\left(\left(1-\mu\frac{d^2}{dx^2}\right).(P.\delta(x-v_p t))\right).dx$$

$$\int_0^L \phi'_i(x).\left(E_{11}\sum_{j=1}^{n}\varphi'_j(x).u_j(t)-F_{11}\sum_{k=1}^{n}\psi''_k(x).w_k(t)+H_{11}\sum_{p=1}^{n}\phi'_p(x).\varphi_p(t)\right).dx+\int_0^L \phi_i(x)\left(A_{55}\sum_{P=1}^{n}\phi_P(x).\varphi_p(t)\right).dx$$
$$+\int_0^L \phi_i(x).\left(\left(I_3\sum_{j=1}^{n}\left(1-\mu\frac{d^2}{dx^2}\right).\varphi_j(x).\ddot{u}_j(t)-I_5\sum_{p=1}^{n}\left(1-\mu\frac{d^2}{dx^2}\right)\psi'_k(x).\ddot{w}_k(t)\right)+I_6\sum_{p=1}^{n}\left(1-\mu\frac{d^2}{dx^2}\right)\phi_p(x).\ddot{\varphi}_p(t)\right)dx+ \quad (31)$$
$$\left[M^{sd}.\phi_i(x)\right]_0^L=0$$

The coefficients of the test functions in limited integrals are called the secondary variables; their specifications are the boundary conditions. The Dirac delta function of the transversal moving load in the equation (30) is defined by [24]:

$$\int_{x_1}^{x_2} f(x).\delta^{(n)}(x-x_p).dx=\begin{cases}-(1)^n.f^n(x_p)\,if\;\; x_1<x_p<x_2\\ 0 \qquad otherwise\end{cases}$$

Where $\delta^{(n)}$ represents nth derivative of Dirac delta function. The secondary variables are turned to zero, the governmental differential equation system of the FGM nanobeam under moving load is written in the following general form:

$$[K]\{q(t)\}+[M]\{\ddot{q}(t)\}=\{F(t)\} \quad (32)$$

[K] and [M] are the stiffness and mass matrices respectively, their order is [3n × 3n], {q (t)} is the column vector of unknown Ritz's temporary coefficients, of the order [3n × 1]. {F (t)} is the generalized vector produced by the moving transverse load of the order [3n × 1].

5 Numerical results and discussions

Dynamic analysis of FGM nanobeams simply supported subjected to forced vibrations is investigated with parameters

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varying: non local parameters (μ), material distribution (k), order of beams theories $f(z)$, slenderness ratio (L/h) and the moving load speed (vp). The analysis results are determined numerically using the Newmark's method with a fixed number of 400 time steps for a desired precision in the calculation.
The mixture is composed of ceramic and metal whose mechanical properties are illustrated in Table 1. The top side of the nanobeam (z = + h /2) is purely ceramic (Alumina), while the bottom side (z = - h /2) is purely metal (Aluminum). The dynamic responses are calculated at the median reach of the nanobeams where the deformations are maximum in this critical section, with one unit of thickness (h = 1nm).

TABLE 1
MECHANICAL PROPERTIES OF THE FGM NANOBEAM

Properties	Unit	Aluminum (Al)	Alumina (Al_2O_3)
E	GPa	70	380
Ρ	kg/m3	2700	3800
υ	-	0.23	0.23

The non-dimensional transverse dynamic deflection D (t) is independent of the material, magnitude of the moving load (P) and the geometric properties of the FGM nanobeams; it is expressed by the following relation:

$$D(t) = \frac{w_d(L/2,t)}{w_s(L/2)}$$

$w_s(L/2)$ is the static deflection, calculated with the material property of the lower surface (P_L) and solicited by the load "P" acting at midspan of the FGM nanobeams, to describe by:

$$w_s\left(\frac{L}{2}\right) = -\frac{P.L^3}{48.E_L.I}$$

The maximum transverse dynamic deflection without dimensions is defined by:

$$D_{\max} = \max(D(t)) = \max\left(\frac{w_d(L/2,t)}{w_s(L/2)}\right)$$

The effects of moving load velocity and moving load passing are plotted by dimensionless parameters (α) and (t*) respectively, as follows:

$$\alpha = \frac{\upsilon_p}{\upsilon_{cr}}, t^* = \frac{\upsilon_p t}{L}$$

Where υ_{cr} is the critical speed defined as [24]:

$$\upsilon_{cr} = \frac{\omega_1 L}{\pi}$$

ω_1: The first fundamental frequency of the FGM nanobeams (rad/s).
Therefore, when $t = 0$, the punctual load (P) is in the left support of the nanobeams $(x_P = 0)$ and when $t = 1$, the point load (P) has arrived at the right support of the nanobeams $(x_P = L)$.

TABLE 2
CONVERGENCE STUDY OF THE DYNAMICS RESPONSES

Number of term (n)	Classical theory			Nonlocal theory		
	CBT	FSDBT	PSDBT	CBT	FSDBT	PSDBT
2	0.4029	0.4029	0.4029	0.4290	0.4290	0.4290
3	0.5582	0.5621	0.5626	0.6262	0.6313	0.6319
4	0.5582	0.5629	0.5635	0.6262	0.6323	0.6331
5	0.5636	0.5680	0.5685	0.6280	0.6340	0.6346
6	0.5636	0.5681	0.5686	0.6280	0.6348	0.6363
7	0.5649	0.5691	0.5696	0.6324	0.6364	0.6372
8	0.5649	0.5691	0.5696	0.6324	0.6374	0.6393

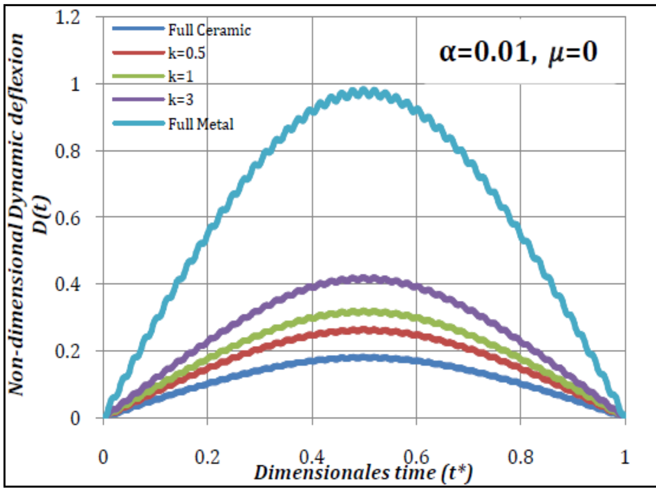
In Table 2, the convergence studies for the maximum transverse dynamic deflection (Dmax) of FGM nanobeam are performed by varying the number of polynomials (n) in the spatial displacement for different shear function (CBT, FSDBT and PSDBT), using two theories, local (μ = 0) and non-local (μ = 4) with k = 3, α = 0.3 and L / h = 20. It's seen that the increase in the number of terms (n) of Ritz polynomials is important in the convergence of Dmax. Arguably, the numerical precision of the maximum transverse dynamic deflection is satisfactory when the number (n) is set to 8 in the displacement function.

TABLE 3
VARIATION OF MAXIMUM TRANSVERSE DYNAMIC DEFLECTION FOR FGM NANOBEAMS

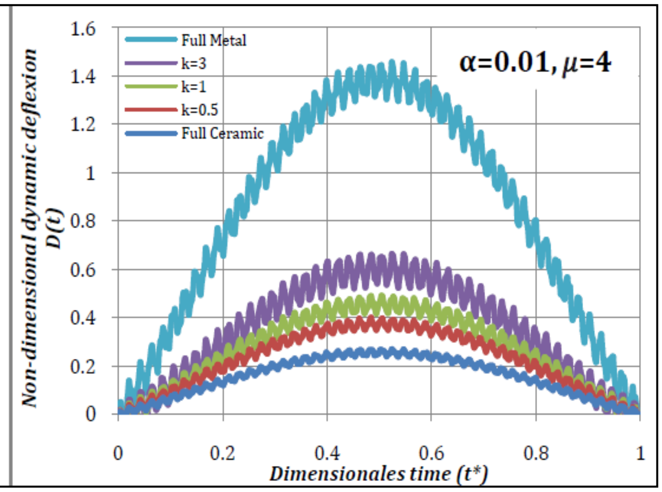
μ (nm^2)	Source	α=0.1	α=0.5	α=1
Local effect μ = 0	Present			
	CBT	1.0952	1.7043	1.5513
	FSDBT	1.1258	1.7520	1.5882
	PSDBT	1.1257	1.7518	1.5881
	Ref. [14]			
	CBT	1.0970	1.7081	1.5481
Nonlocal effect μ = 4	Present			
	CBT	1.5833	2.4113	2.1688
	FSDBT	1.6790	2.4519	2.3096
	PSDBT	1.6779	2.4528	2.3088
	Ref. [14]			
	CBT	1.6024	2.4146	2.1655

In Table 3, the non-dimensional maximum transverse dynamic deflection (Dmax) of simply supported FGM nanobeam is calculated with the corresponding velocities relative to all order beam theories (CBT, FSDBT and PSDBT), for the nonlocal parameters (μ = 0 and 4) with k = 0, and compared to that obtained by Simsek in Ref. [14]. The length (L) of the nanobeam is assumed to be 10 nm and the Poisson's ratio (υ) is taken as 0.3. The results in this table are presented without taking into account the effect of the Poisson's ratio (υ) in the expression of the reduced stiffness coefficient (Q11).
The comparisons show an excellent agreement between the results that validate the accuracy of our developed model. However, the maximum transverse dynamic deflections (Dmax) calculated by considering the shear effect (FSDBT and PSDBT) are relatively higher compared to those calculated with classical Euler-Bernoulli beam theory (CBT), this difference is greater

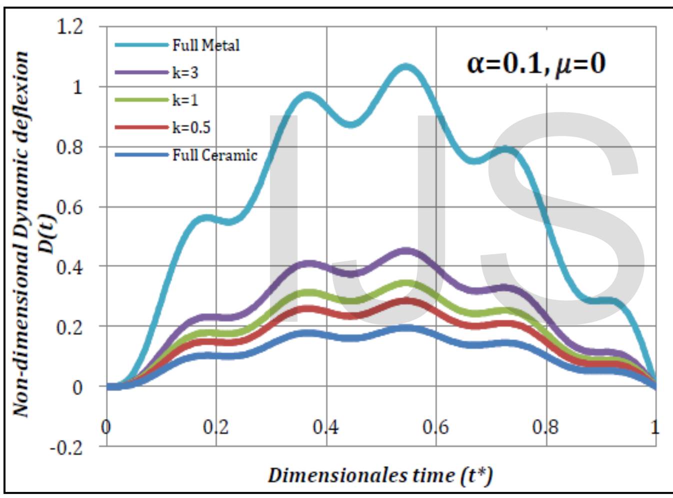
when the nanobeam is modalized with the nonlocal elasticity (μ = 4), while the two theories FSDBT and PSDBT give substantially the same results, for different nonlocal parameter (μ=0 and 4). This implies that the shear deformation beams effect must be taken into account in the calculation for the dynamic deflection of FGM nanobeams.



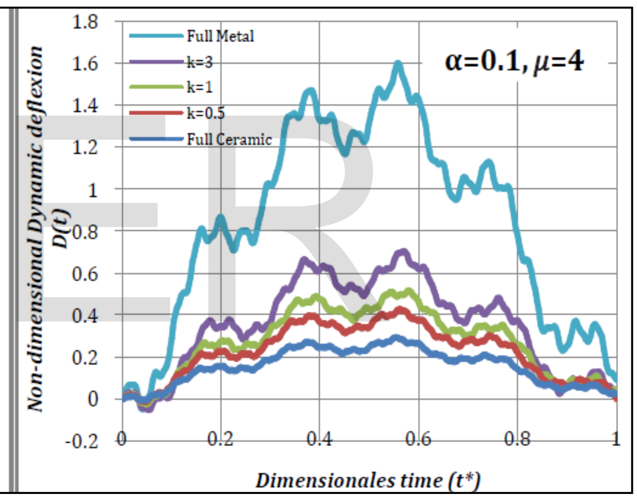
(a-1)



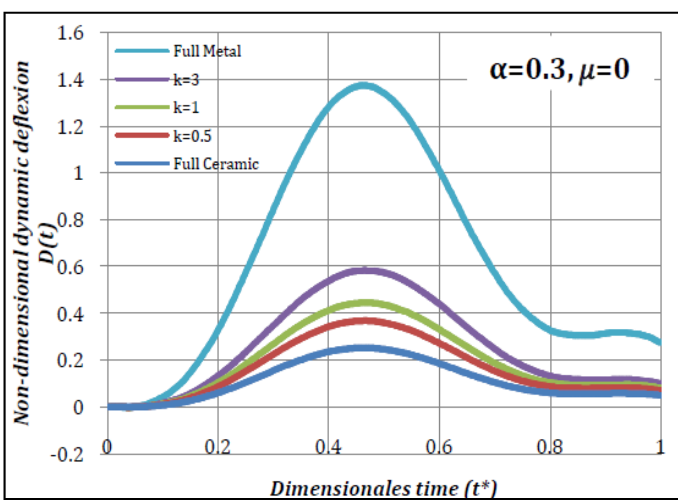
(a-2)



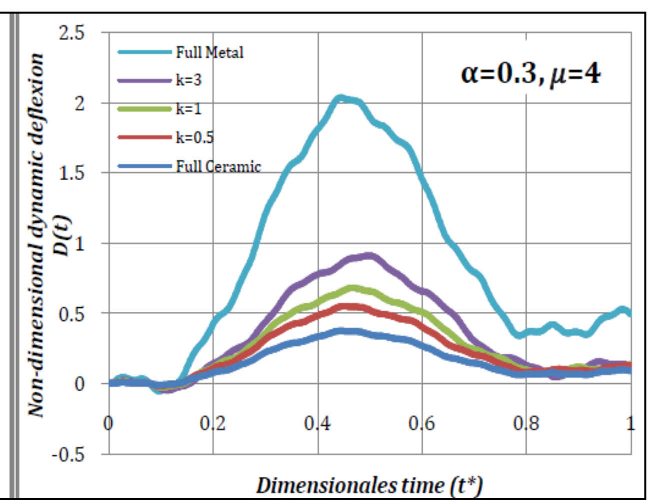
(b-1)



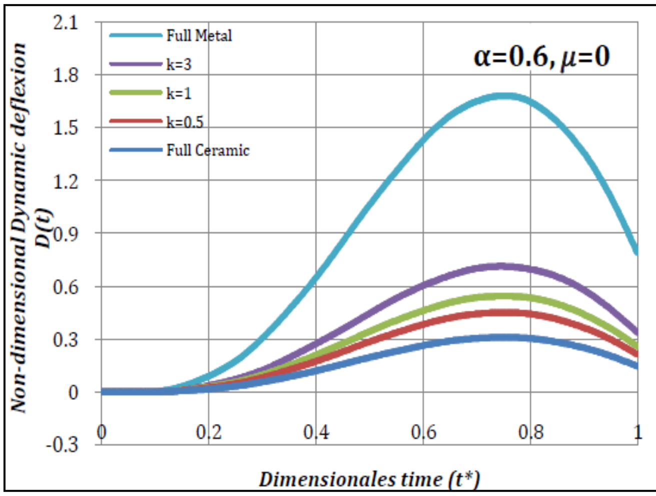
(b-2)



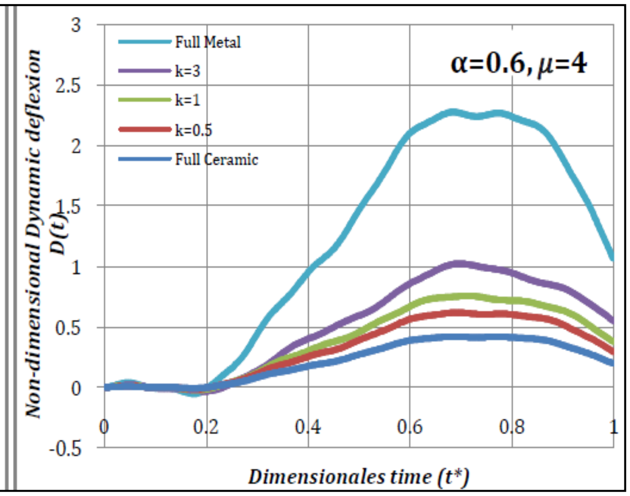
(c-1)



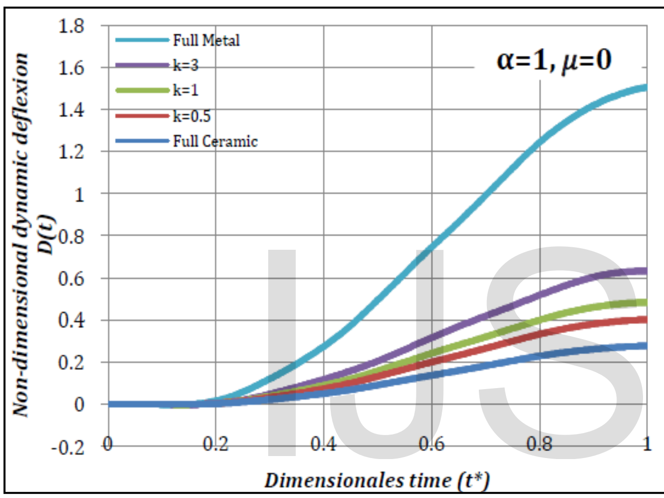
(c-2)



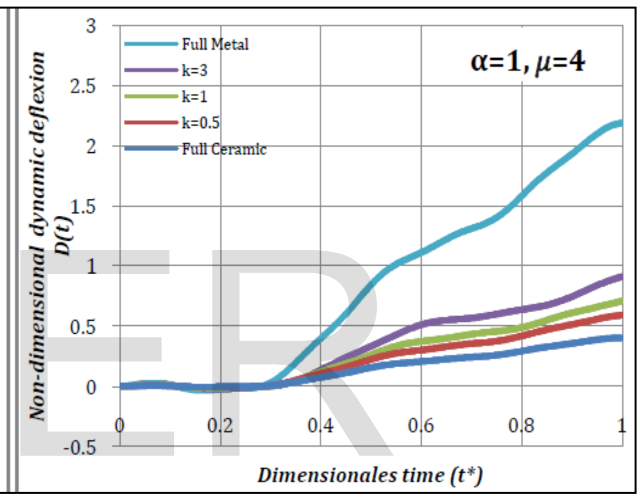
(d-1)



(d-2)



(e-1)



(e-2)

FIG.3. TIME HISTORY OF THE MIDSPAN DISPLACEMENTS

These figures show the temporal history of the non-dimensional transversal dynamic deflection of the midspan FGM nanobeam modeled by the high-order shear deformation beam theory (PSDBT), for different values of material distribution parameter (k) using two different theories, local (classical) and nonlocal. This deflection is traced under various dimensionless velocities of the moving load (α=0.01, 0.1, 0.3, 0.6 and 1).

It's noted that the dynamic deflections are proportional to the power law exponent (k), these values become maximal when the nanobeam is made in pure metal ($k \rightarrow \infty$), this increase of the response is due to the increase in amount of metal in the mixture which results in a decrease in Young's modulus and flexural rigidity. We also note that the dynamic deflections obtained using nonlocal theory is much higher to that of classical theory (local) because of the small scale effect.

It can also be observed that the moving load velocities plays a crucial role on the vibratory behavior of the FGM nanobeam, the critical location of the moving force which does given a maximum dynamic deflection changes with the values speed, it is very significant when α is on the margin of 0.3 to 1 where the maximum deflection moves in the middle towards the right end of the nanobeam. For low velocities (α = 0.01), the dynamic deflection reacts similarly to a quasi-static nanobeam figure (a). Consequently, the reduction of moving load speed gives a flexibility to the system which itself generates an instability on the form of dynamic responses, it is remarkable in the nonlocal theory which has a low flexural rigidity (figures a-1 and b-1). When the load moves at a faster speed, the FGM nanobeam does not have enough time to respond to the force that gives us stability in the system and the two theories converge on a single mode of vibratory behavior as indicated by figures c, d and e.

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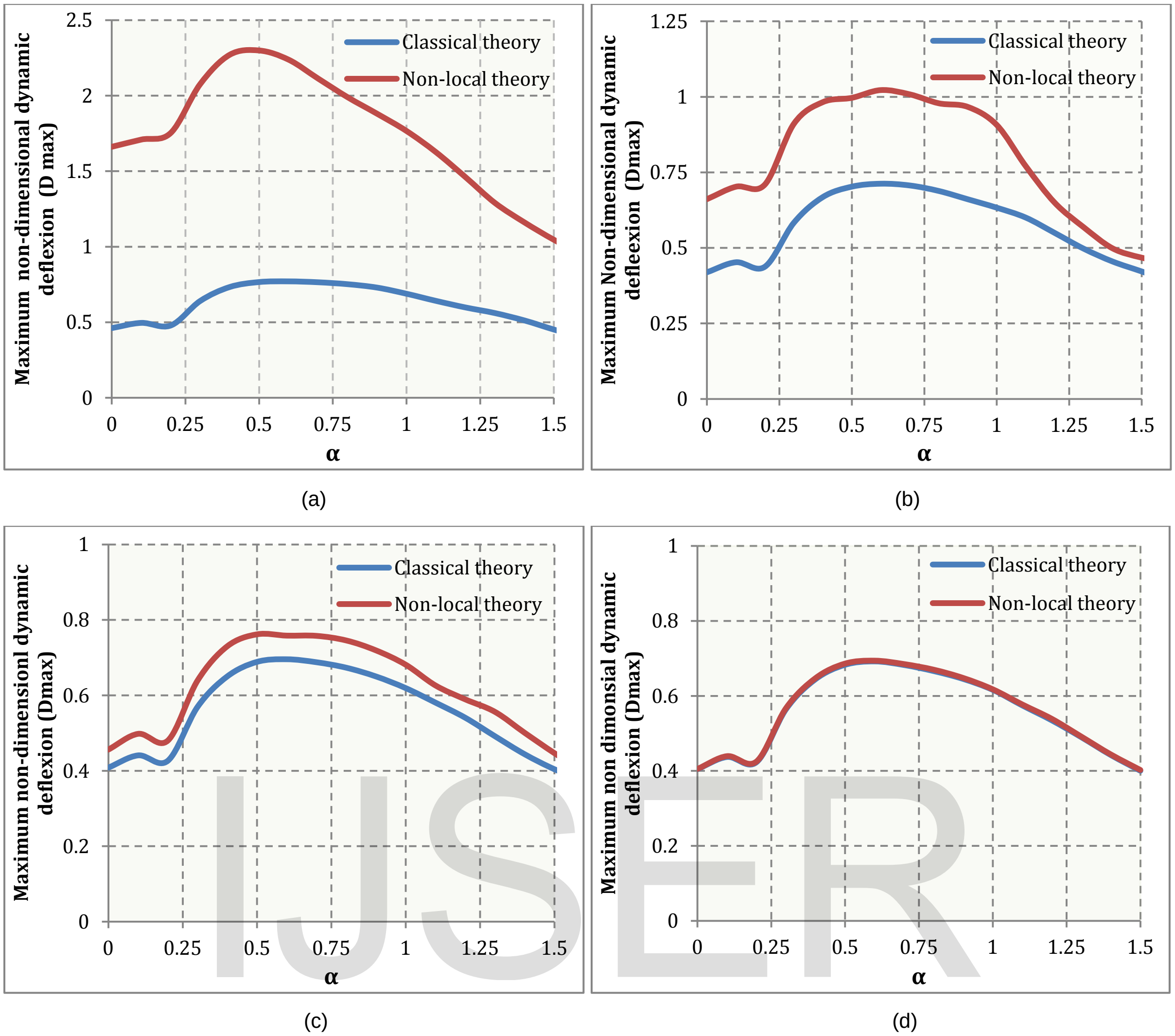

FIG.4. VARIATION OF THE MAXIMUM NON-DIMENSIONAL TRANSVERSE DYNAMIC DEFLECTION OF THE FGM NANOBEAMS

Figure (4) shows the relationship between the velocity of the moving load (α) and the maximum dimensionless dynamic deflection (Dmax) at the center of FGM nanobeam modeled by the Parabolic Shear Deformation Beam Theory (PSDBT), with the power law exponent k = 3, using two theories, local (classical) and nonlocal, for different slenderness ratio values (L / h = 5, 10, 20 and 100). However, it is observed that the velocities of the load affects considerably on the maximum dynamic deflection; the amplitude has the highest rate when the applied speed is in margin of 50% to 60% of the critical speed, after this value, the increase of the velocities involves a reduction of the maximum dynamic deflections. For a dimensionless speed parameter (α) is set to 0, this is the case of the static deflection loaded with a concentrated force in the middle of the FGM nanobeam.

In addition, the impact of the slenderness ratio (L / h) is inversely proportional to the maximum dynamic deflection; this influence is more significant when the nonlocal parameters is included in the vibratory analysis; for example, for an aspect ratio (L / h) increase from 5 to 20, the highest rate of maximum dynamic deflection (Dmax) decreases by 66.86% for the nonlocal theory and a slight decrease of 09.76% in the classical theory (local).

It should also be noted that the importance of the scale effect on the maximum dynamic deflection becomes less obvious when the FGM nanobeam start to take a higher values of slenderness ratios; for example, for L / h = 5, the highest rate of Dmax modeled by the nonlocal theory is 198.13% superior than the highest rate of Dmax modeled by the classical theory, 43.56% for L / h = 10 and 09.49% for L / h = 20. For very thin FGM nanobeams (Figure 4d), the nonlocal effect on maximum dynamic deflection is negligible.

6 CONCLUSION

In this paper, an analytical model is presented to study the dynamic behavior of FGM nanobeam traversed by a constant moving load, based on the constitutive relationship of Eringen, with using various order beams theories. The motion equations are defined by the Hamilton principle with introduction of weak forms in the analysis. The shape functions indicating the displacements of the FGM nanobeams are expressed in a polynomial series of Ritz and the Newmark method is adopted to solve numerically the governmental equations of the forced vibratory system. Numerical results are validated with those available in the literature. The effects of different parameters are also examined in detail. The main conclusions of this investigation are:

- ✓ The increased number (n) in the displacement function plays a major role in the convergence of dynamic responses; which favors the Ritz polynomial in the structures and nanostructures programming.
- ✓ Because of the small-scale affect, the dynamic deflections obtained by using the nonlocal theory are always greater than that obtained by the classical (local) theory.
- ✓ The dynamic responses are proportional to the power exponent (k) of the FGM function, which in turn is inversely proportional to Young's modulus and nanobeam rigidity.
- ✓ The transverse dynamic deflections calculated considering the shear beam effect (FSDBT and PSDBT) are relatively higher compared than those calculated with Euler-Bernoulli theory (CBT).
- ✓ The critical position of the moving punctual load that corresponds to the maximum value of dynamic deflections is significantly influenced by the applied velocity.
- ✓ The decreasing in the speeds value of the moving load generates instability on the dynamic behavior mode of the vibratory system; these oscillations are very cruel in structures that have a low flexural stiffness with consideration of the small-scale effect.
- ✓ The transverse dynamic deflections found by local theory are almost independent of the slenderness ratio compared to nonlocal theory.
- ✓ The scale parameters greatly affect the dynamic deflections of the nanobeams that contain a lower slenderness ratio.
- ✓ The nonlocal parameter, shear deformation, material distribution, slenderness ratio and moving load velocity have a decisive impact in the dynamic response analysis of FGM nanobeams subjected to forced vibrations.

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